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THE PROBLEM OF BRITTLE FRACTURE IN METALS

N.J. Addox Pieter Hinkamp

Lieut. Comdr., U.S.N.

March, 1948

Thesis H672H

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M.R. Pieter Hinkamp

THE PROBLEM OF BRITTLE FRACTURE IN METALS

Attention has been focused in recent years on the all-important subject of brittle fracture. It is well to point out that if there were no controversies or discrepancies in the theories concerning brittle fracture, and if the state of knowledge was in a fairly satisfactory condition, the serious epidemic of failures in welded ship structures during World War II might never have happened. Much of the recent investigation and attention given to this subject stems from this one series of failures as added impetus towards the solution of one of the most outstanding questions in metallurgy.

This paper will be confined to the discussion of failure in the brittle manner only, and will not attempt to go into the field of ductile failure. It is the author's intention to present in concise form both the basic fundamentals and the most modern thought on this important subject, as well as some of the currently controversial points. A discussion of the modern theories concerning brittle fracture will be preceded by a brief analysis of the stress system acting at the moment of fracture. The effect of various factors on brittle fracture will be shown later.

There are four approaches to the problem of ~~the problem of~~ fracture, namely;-

a. The study of the overall features, as evidenced by the

THE PROBLEM OF BUCKLING PREDICTED IN CONCRETE

Attention has been focused in recent years on the all-important subject of buckle fracture. It is well to point out that if there were no contradictions or discrepancies in the theories surrounding buckle fracture, and if the state of knowledge was in a fairly satisfactory condition, the serious epidemic of failures we related silo structures during World War II might never have happened. Some of the recent investigations and attempts given to this subject stems from the series of failures as noted therein towards the solution of one of the most outstanding problems in metallurgy.

This paper will be confined to the discussion of failure in buckle welder only, and will attempt to go into the detail of buckle failure. It is the writer's intention to present an acceptable form both the basic fundamentals and the most modern thought on this interesting subject, as well as some of the controversial points. A discussion of the modern theories concerning buckle fracture will be preceded by a brief analysis of the stress system existing at the moment of fracture. The effect of various factors on buckle fracture will be given later.

There are four difficulties in the problem of ~~the prediction of~~ fracture, namely:-

a. The study of the overall features, as evidenced by the

stress system acting, in an effort to determine the conditions under which the fracture will occur.

b. The study of the metallurgical and chrystallographic structures to determine fracture characteristics.

c. The study of fractures from the level of atoms and cohesive energies.

d. The study from the standpoint of thermodynamics.

Definitions:-

Before approaching the discussion of the stress system, definitions of brittle and ductile fracture should be made in the interests of clarity. In either case, a broad definition of fracture says that it is the termination of plastic flow in which there is a separation of surfaces. Ductile fracture occurs after a relatively large amount of plastic flow by the mechanism of shearing along the slip planes, which are generally at an angle of 45' to the axis along which the maximum normal stress is acting. The appearance of the fracture is characterized as fibrous, dull, and rough. Brittle fracture occurs after little or no plastic flow, occurs along cleavage planes within the metal crystals, and is characterized by being normal to the axis along which the maximum normal stress is acting and is bright and crystalline in appearance due to the many tiny facets produced when the crystals are cleaved. Fractograph work by Zapffe on Bi, Zn, and Sb illustrate cleavage facets at high magnifications.

~~Mechanism of Plastic Flow~~:-

Mechanism of Plastic Flow:-

Since by the above definitions it can be seen that fracture is inseparable from plastic flow, the mechanism of plastic flow should be briefly stated in order to fully understand the discussions relating to brittle fracture. Plastic flow occurs either by slipping or by twinning. Most of the experimental evidence is with single crystals in an effort to avoid many complicating factors. Slipping will cause a reorientation of areas within single crystals due to different slip directions and, of course, a reorientation of whole crystals. When grain bounderies are interposed between crystals, flow is interrupted by the conflicting slip directions and inhomogeneous strains are set up within the metal. This is the chief obstacle to the study of plastic flow in crystal aggregates. It has also been found that inclusions will reorient themselves to the slip direction and that fatigue or alternating stresses will cause slip bands to widen when plastic flow has been initiated. Crystal aggregates generally offer more resistance to flow than do single crystals. A further interesting concept is that of the viscosity or the behavior in the manner of an amorphous material in the grain bounderies. It appears that this is a transition region where the atoms for a few layers are not arranged in the crystalline structure of either grain. Slip bands also exhibit this amorphous behavior under strain.

Analysis of the Stress System:-

As an introduction to the analysis of the stress system, it is useful to assume that the amount of deformation that can occur

before flow or fracture sets in is determined by the relative values of the stresses required to cause slip, twinning, and cleavage. The recognized interrelation between plastic flow and brittle fracture brings up the added factor of the anisotropy introduced by the plastic strain. Non-perfect isotropy will invalidate the agreement with the following theory of combined stresses, but it is to be noted that at the small strains associated with brittle fracture the anisotropy is not great. Now, in single crystals, it is known that plastic flow will initiate when the stress, as resolved on the slip plane and in the slip direction, reaches a critical value(the critical shear stress). In polycrystalline metals, however, it is believed that plastic flow occurs when the shear strain energy reaches a critical value. This concept may be expressed by the following equation in which (σ_o) is the critical value and (σ_1), (σ_2), and (σ_3) are the principal stresses along the principal axes (1),(2), and (3):

$$2\sigma_o^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

It is known that this equation is good for determining the beginning of plastic flow for most metals which will yield homogeneously. This equation of Von Mises has been substantiated for the cases of uniaxial and biaxial tension but not for the case of triaxial tension,due to the experimental difficulties and to some uncertainties surrounding the use of the notch bar impact test as a criterion. In the case of metals which yield inhomogeneously, the maximum shear stress law seems to be

obeyed in that yielding will occur when the maximum shear stress, $\frac{\sigma_1 - \sigma_3}{2}$ reaches a critical value. A complication in this argument is introduced by Guest in which he proposes a theory that takes into account volumetric stresses. Experimental evidence in the form of adding hydrostatic pressure to the normal tensile stresses seems to validate his argument. This is but one of the many contradictory arguments surrounding this subject.

It should be noted here that the above discussion pertains to an idealized material and thus will only apply strictly to a small portion of the stress-strain curve for which values are known for the principal stresses. There is an extension of Von Mises' theory derived by Hencky, which states that the second invarients of the stress and strain tensors are functionally related. Ilyushin has extended this principle and shown case solutions. Prager advises caution in the use of these power laws, pointing out important variances between solutions and actual test plots. It is not within the scope of this paper to derive or prove this statement but merely to indicate the existance of this powerful tool. It can be said that the intensity of stress (σ_i) defines a quantity whose square, except for a constant factor, is equal to the second invarient of the stress deviation. The intensity of strain ($\mathcal{E}_i$) is defined in a similar manner. The above values are sometimes also known as the significant stress ($\bar{\sigma}$) and the significant strain ($\bar{e}$) and are written as follows:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_1 - \sigma_2)^2 - (\sigma_2 - \sigma_3)^2 - (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}}$$

$$\sigma_i = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

$$\bar{e} = \sqrt{\frac{2}{3}} \left[e_1^2 + e_2^2 + e_3^2 \right]^{\frac{1}{2}}$$ where e_1, e_2, e_3, are the strains along the principal axes.

$$\bar{e} = D\,\bar{\sigma}, \text{ or } \bar{\sigma} = k\bar{e}^n$$ D, K, and n, are constants. $0 < n < 1$

These equations give a generalized stress-strain relationship which does not depend on the type of stress.

There are two other important relationships usually assumed in problems of plastic flow. These are;

a. The material is incompressible/ or the change in volume during plastic flow is zero.

$$e_1 + e_2 + e_3 = 0$$

b. The principle shearing stresses and strains are proportional:

$$\frac{\sigma_1 - \sigma_2}{e_1 - e_2} = \frac{\sigma_2 - \sigma_3}{e_2 - e_3} = \frac{\sigma_3 - \sigma_1}{e_3 - e_1}$$ (Hooke's Law)

With the generalized stress-strain relationships noted above, it is important to note that the significant stress and the significant strain are related by the proportionality factor (D). This relationship may take several convenient forms, not herein noted, for purposes of calculating specific problems.

Idealized Stress-Strain Diagram:-

The previous discussion leads to the stress-strain diagrams as determined for idealized materials and from which aid in further discussion of modern theories and effects of various factors on brittle fracture may be had. It is sufficient to merely

$$\bar{e} = \sqrt{\frac{2}{3}}\left[e_1^2 + e_2^2 + e_3^2\right]^{\frac{1}{2}}$$

where e_1, e_2, e_3 are the

principal values of the devia-

toric strain tensor.

$$\bar{\sigma} = \sum \sigma_i' \sigma_i = \bar{\sigma}\bar{e}_w \qquad 0 < w < 1$$

σ_i', e_i' and w are constants.

There exist other important relationships resulting assuming plastic flow to depend on the rate of stress. These relationships might does not depend on the rate of stress.

In practice of plastic flow. These are:

a. The material is incompressible, or the change in volume during plastic flow is zero.

$$e_1' + e_2' + e_3' = 0$$

b. The principle stresses and strains are proportional:

$$\frac{e_1' - e_2'}{\sigma_1' - \sigma_2'} = \frac{e_2' - e_3'}{\sigma_2' - \sigma_3'} = \frac{e_3' - e_1'}{\sigma_3' - \sigma_1'} \qquad \text{(Lévy's law)}$$

(the generalized stress-strain relationship noted above. It is important to note that the principal stress and the principal shear strain are related by the proportionality factor (2). This relationship can take several equivalent forms, not herein noted, for purposes of mathematical analytic treatment.

Plastic Stress-Strain Diagrams:

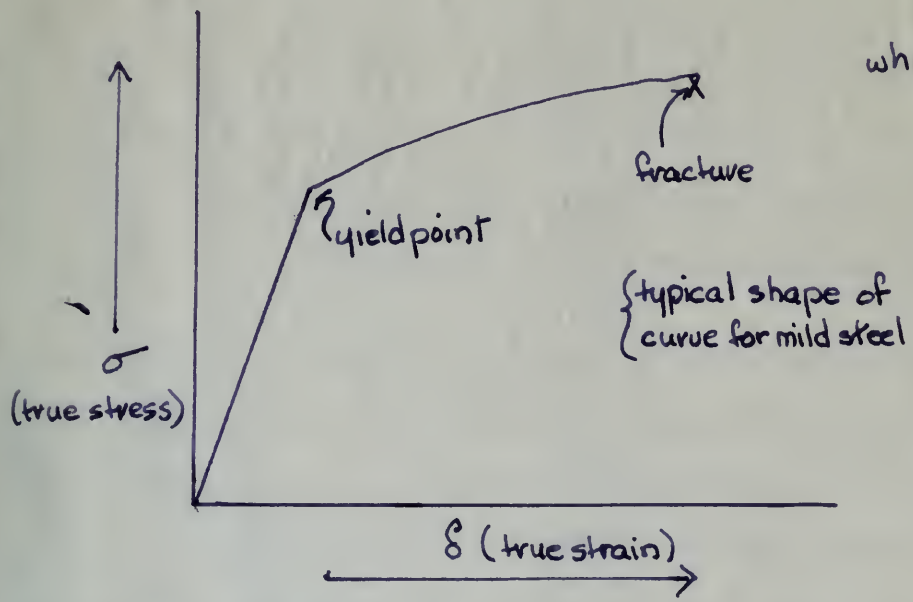
The previous discussion leads to the stress-strain diagram for an ideal rigid-plastic material and also within all its future discussion of second kinetics and effects of multiple dimensions. It is sufficient to note that a sufficient is merely zero to initial tension and to that. It is sufficient to merely

note the nominal stress-strain curve in which load/ original cross-section area (tensile specimen) is plotted against nominal strain. This type of curve gives rise to the familiar drop of the beam shape as experienced with steels. A modification of this early curve is the plot of true stress, load/ actual cross-section area at the instant of measurement, vs true strain, which is expressed as follows:

Since $e_1 + e_2 + e_3 = 0$ (or the volume remains constant)

$$Al = A_0 l_0 \; ; \; \delta = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} = \ln \frac{A_0}{A} = \ln \frac{d_0^2}{d^2} = 2 \ln \frac{d_0}{d}$$

where d = diameter
A = cross-section ave
l = length
d_0 = original diame
δ = true strain



The above type of curve is known as the true stress-strain curve or more properly as a flow curve. The mathematical expressions previously noted generally apply well to the elastic region, but, where the problem of necking is encountered, the complex stress system and the non-uniform deformation cause the situation to be immeasurably complicated.

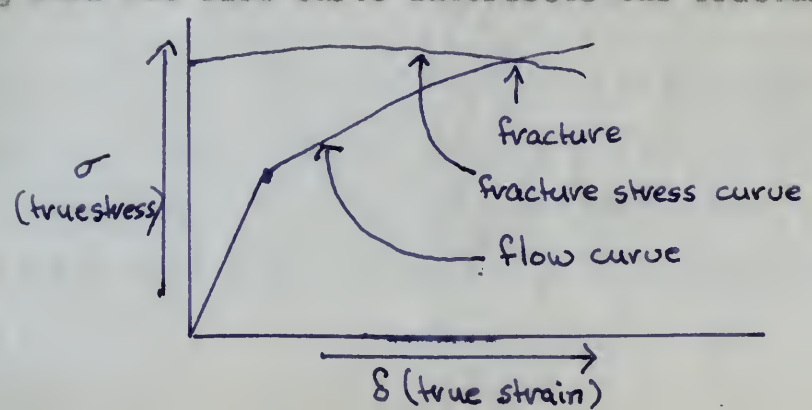
Since $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ (or the volume remains constant)
$Al = A_0 l_0$; $\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} = \ln \frac{A_0}{A} = \ln \frac{d_0^2}{d^2} = 2 \ln \frac{d_0}{d}$
where
d = diameter
A = cross-section area
l = length
d_0 = original diameter
ε = true strain
fracture
yield point
(typical shape of curve for mild steel)
(true stress)
ε (true strain)

The above type of curve is known as the true stress-strain curve or more properly as a flow curve. The mathematical expressions previously noted reasonably apply well in the elastic region, but where the problem of locating the proportional limit remains there is the non-uniform deformation curve for alternate to be measurably complicated.

Ludwik's Theory:-

In beginning the discussion of the various theories, the introduction of the flow curve above leads to the discussion of Ludwik's theories and his postulation of a fictitious fracture curve. His work has been the basis of a great deal of research and thought on this matter particularly by Hollomon in recent years. The flow curve, per se, was originally recognized by Ludwik as possessing great possibilities for the interpretation of the mathematical statements. It is to be remembered that this curve applies to idealized material and thus requires a correction factor if it is to be used directly for any real material. Several people, notably Bridgeman and Davidenkov, have attempted to find suitable corrections to bring the plastic region of the curve down to meet the actual curve as determined from experiments. They have been more or less successful.

Ludwik conceived the idea of a fracture stress curve which is based upon saying that the fracture stress just before a metal fractures is greater than the stress required for plastic flow and that this fracture stress is dependent upon the prior deformation which the unbroken metal had gone through. It can be seen that a fracture stress and a flow curve must be determined under different external conditions. Ludwik postulates fracture as occuring when the flow curve intersects the fracture curve as seen;



Griffith's Theory-

In applying the discussion of the various theories, the interpretation of the flow curve above leads to the discussion of Griffith's theories and his postulation of a fracture curve. Theoretically he used the basic principles of energy and thought on this matter particularly by Inglis in assessing great possibilities for the interpretation of the mathematical relationship. It is to be remembered that this curve applies to idealized material and thus requires a correction factor if it is to be used directly for any real material. Several people, notably Orowan and Davidenkov, have attempted to find suitable corrections to bring the plastic region of the curve down to meet the actual curve as determined from experiments. They have been more or less successful.

Griffith considered the idea of a fracture stress curve which is based upon saying that the fracture stress just before a metal fractures is greater than the stress required for plastic flow and that this fracture stress is dependent upon the polar distribution of the microscopic metal set upon stressing. It can be seen that a fracture stress and a flow curve must be determined under different external conditions. Griffith postulates fractures as occurring when the flow curve intersects the fracture curve as exemplified in the following diagram:

The greatest justification for the use of a fracture curve, although it is a fictitious concept, is its inherent ability to explain conveniently many different effects on the fracture of metals. The various effects are noted as raising or lowering the flow curve, the fracture curve, or both.

Sollomon has pointed out that there may be a great effect due to anisotropy of the metal structure as caused by prior deformation. This point has not been sufficiently investigated. Fracture stress analyses made with notched tensile bars have been recently made by McAdam, in which it is assumed that fracture begins in the center of the bars. Results can be interpreted as applying triaxially in which fracture stress is shown to increase with increasing triaxiality. This is further coöberated by experiments performed by Sachs. In this case, the fracture curve rises, but, at the same time, the flow curve is also rising at a greater rate, which essentially reduces the strain needed for fracture although the stress is raised. This serves as one illustration of the use of flow and fracture curves. There has been a great deal of ~~criticism~~ criticism of these curves, but it is desired to emphasize that they make no attempt to explain the whole phenomenon of fracture but serve as a useful tool and as such should not be overlooked. It should be further emphasized that the fracture curve is a purely fictitious concept.

Sternberg has replaced the linear expression of Hooke's law, previously noted, with a general second order approximation which assumes non-linearity within the elastic range. His results show definite second order effects, in particular, where such

materials as cast iron and concrete are concerned. Sternberg's work is based on Voigt's five parameter theory in which the classical expression of Hooke's law is the limiting case. It is believed that this theory of non-linearity within the elastic range may well explain the baffling behavior of cast iron according to conventional solutions.

Micro-Crack Theory:-

Strength, as applied to metals has a very definite meaning:

a. Resistance to flow.

b. Resistance to fracture.

The fracture strength is very much more structure sensitive than the flow strength, which is an important factor when considering the capacity to deform and to absorb energy before fracture. Closely related to the above is the structure sensitivity of the cohesive strength in which it is known that minute imperfections or even a precipitate will induce brittle behavior. This introduces the well known micro-crack theory.

One of the most intriguing aspects of the whole problem of fracture and the one which may hold the key to the ultimate explanation of the phenomenon is the effort to explain why the actual strength of metals is so low when by calculations based on the cohesive strength between atoms and lattice planes the fracture strength should be from one hundred to one thousand times greater. In a general way, the micro-crack theory attempts to explain this great discrepency by saying that there are tiny cracks within the crystal structure of the metal which act as stress raisers and thus concentrate the stress under conditions

of restraint to the point that the fracture strength, in a highly localized region, is exceeded and fracture initiates at a stress very much lower than that predicted by the cohesive strength of the atomic bonds.

Griffith has worked on this problem from an interesting point of view and his ideas have formed the basis of a great deal of recent work. His fundamental concept is that, in a solid, the boundary surfaces possess a surface tension which implies the existance of a corresponding amount of potential energy, and if, due to stress, a crack is formed or an already existing one is extended, an amount of energy proportional to the area of the new surface formed must be added and this must be done without any increase in the total potential energy of the whole system. This new energy must come from the decrease in the potential caused by the spread of the crack. These two energies must be balanced for the crack to propagate. Based on this reasoning, Griffith derived an expression for the fracture stress:(σ_f):-

$$\sigma_f = \sqrt{\frac{2SE}{Cx}}$$

Where---

- S is the surface energy/ unit area
- E " " modulus of elasticity
- x " " length of the crack
- C " " constant

There are three assumptions that should be mentioned;

1. The material is perfectly elastic.
2. The distribution of cracks is such

of resistant to the point that the fracture appears, in a stig-
ly localized region, is exceeded and fracture initiates at a
stress very much lower than that predicted by the cohesive
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energies must be balanced for the crack to propagate. Based on
this reasoning, Griffith derived an expression for the fracture
stress: (σ_f):-

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi c}}$$

where---

- σ is the surface energy/ unit area
- E " modulus of elasticity
- $2c$ " length of the crack
- π " constant

There are three assumptions that should be mentioned:

1. The material is perfectly elastic.
2. The distribution of cracks is even

that there is no mutual interference.

3. The state of the stress is two-dimensional.

In using the above equation it is further assumed that the cracks are plane discs, perpendicular to the axis of the specimen.

Griffith carried out his experiments using glass rods of varying thicknesses. With extremely thin glass rods he was able to attain a strength of about one fourth the theoretical but when he used the slightly larger rods his strength fell off rapidly. This brings up the size effect which will be discussed later. His ideas relative to the effect of defects in causing fracture can, however, be applied to metals. Assuming the presence of the defects, when a metal is strained in the plastic region, the defects will reorient themselves if they are plate-like so that less strain energy will be released by their propagation and the stress at which fracture will occur will be raised. Another concept is that, as the defect is rotated away from being perpendicular to the axis of stress, the stress concentration at the end of the defect becomes less and the defect then has proportionally less effect on lowering the fracture stress.

Zener theorizes that the grain bounderies of metals are perfectly homogeneous but have less resistance to slip than the interior of the grain. When a strain is applied, a stress concentration is set up in the grain bounderies due to the inhomogeniety of the strain and he then believes that this may lead to sufficient internal energy to propegate cracks. It should be remembered that the defect causing fracture is the one producing the largest stress concentration.

There are criticisms of the micro-crack theory which should be noted;

1. It must be assumed that the weaknesses occur in a fairly regular fashion.
2. If stress concentrations are involved in the fracture process they could not alone account for the low shearing stress values for slip.

Statistical Theory:-

There are several observed facts which are fairly well known and which have led to additional research, some of it along statistical lines. With steels, the component of the metallurgical structure that controls the fracture stress is the size, shape, and distribution of the carbide particles. When the carbides are speroidized, for example, the fracture stress is considerably raised, due to the lowering of the stress concentrations. It is also known that the fracture stress depends greatly upon the prior deformation or strain and that the shape of the fracture curve varies widely.

The size effect, as previously noted, in the work of Griffith, has a very important bearing on the fracture strength. The observed facts are that, as the specimen size is increased, brittle fracture will occur with less unit stress and with less work per unit volume required to propagate the crack. These facts led to the statistical analysis by Ruark of crack distribution and density as well as the study of the number, type, and distribution of inclusions per square inch of etched surface on steel.

There are variations of the electro-dynamic theory which should be noted.

1. It must be assumed that the resistance about in a fairly regular fashion.

2. If strong condensations are involved in the theories presumed that could not alone account for the low observed values they aid.

[illegible] Theory:-

There are several observed facts which any fairly well known and which have led to additional researches, some of it along wire-lated lines. With steels, the occurrence of the ferromagnetic [illegible] that explain the Pfaffers steels is the case, among many characteristics of the magnetic particles. When the capacities are accelerated, for example, the Pfaffers wires is negligible ably veiled, due to the lowering of the strong resonations. It is also known that the Pfaffers wires decrease greatly upon the prime determination or alloys and that only of the Pfaffers more veiled clearly.

The wire wires, as previously noted, in the work of Pfaffer, are a very important bearing on the Pfaffers strength. The observed facts are that, as the magnetic wire is increased, unfair fraction will begin with less until minus and wire less work pa- ble volume required to complete the remark. These facts led to the wireless-[illegible] analysis by means of wave distribution and density as well as the study of the number, type, and distribu- tion of locations for magnetic lack of wanted surfaces or wires.

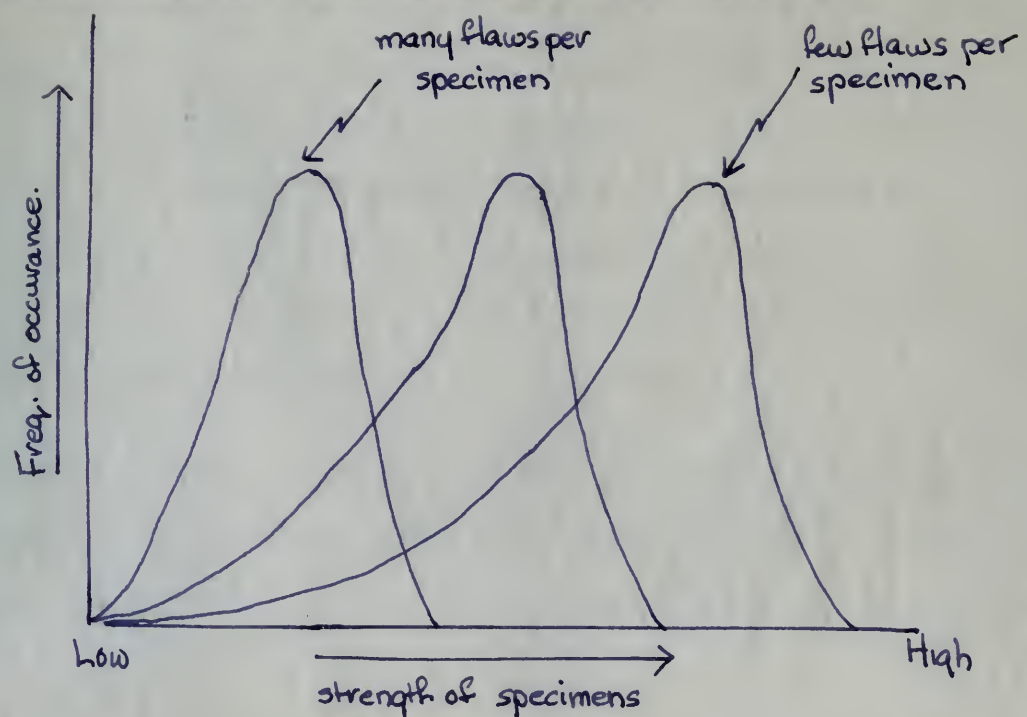
An interesting comment in Ruark's work expresses the belief that the influence of the general pattern of stress raisers is the most potent factor available for explaining otherwise incomprehensible variations in the behavior of different heats of steel. Davidenkov, in working with plain carbon steels, found that a threefold increase in all dimensions does not give a three-fold increase in the energy required to break under impact but rather only about half the expected value. He further found a greater tendency towards brittle fracture with large specimens. He showed that the size effect is not connected with the velocity of testing and thus he based his experiments on the notch bend test rather than the impact test. Davidenkov stated that statistical theory is the only way to explain the size effect and he based his work on the statistical mathematics of Weibull, who established the dependence of strength on the volume of the specimen.

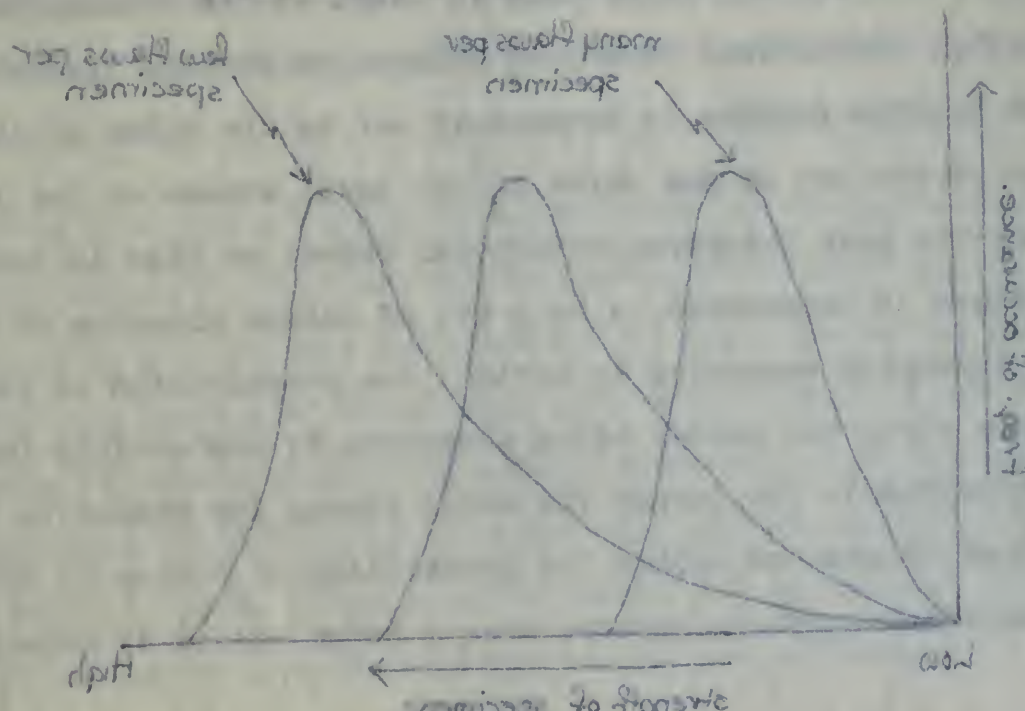
Weibull pioneered the statistical approach to the problem of fracture and all later work is based on his fundamentals. Briefly, statistical theory as related to brittle fracture says that brittle failure is determined not by the value of the average stress but by the value of the local stress at the locus where the most dangerous structural defect or flaw is located. The specimen is considered to be a set of volume elements of varying strengths connected in series, the distribution of strength values along the series being according to probability (random distribution). The larger the whole piece, the weaker is the weakest element or rather the probability of having an extreme value of strength is greater when a large number of elements

is used. Weibull noted that the dispersion in measured tensile strengths required a distribution function to measure it. The statistical problem is the one of distribution of the smallest value in samples of size (n) drawn from a population having some probability density function, f(x). The probability of rupture (S) at any given distribution of stresses (σ) over a volume(V) is:

$$\log(1-S) = -\int_V n(\sigma)\, dv$$

where n (σ) is a characteristic function of each material. This allows for a calculation of the effect of volume on the tensile strength. It is to be noted that the flaws are assumed to be randomly distributed through the specimen and that each flaw is independent of the other. This theory is good for explaining brittle fracture only, and even then makes no effort to explain the entire phenomenon. The following curve shows qualitatively how the strength of the specimen varies with the number of flaws per specimen and their frequency of occurance;




Few flaws per specimen
Many flaws per specimen
No. of occurrences
Low
High
Strength of specimens

Fisher and Holloman have pursued this statistical trend in an effort to show the effects of strain rate, size, and structure. " The fracture of a real material can be investigated theoretically only by idealizing both the strugture of the material and the method of fracture. The idealized model selected for analysis can be justified and accepted as a valid representation if the results of the theoretical analysis are in agreement with the obsrved facts...."

The above quotation characterizes this work, which again is based on Weibull's fundamental statistical analysis. Thair assumptions are:

1. The structure is an elastic solid containing many cracks.
2. The cracks are thin and disc-like with elliptical cross sections.
3. The cracks are orientated at random.
4. The cracks are separated on the average widely enough so that there is negligible amount of interference of the regions of local distortion which surround each crack under elastic strain.

The mathematics will not be presented within this paper but it is sufficient to point out that the normal laws of probability and statistical analysis were followed.

The results of these very interesting statistical analyses into brittle fracture give means to calculate the effect of specimen size on the fracture stress, predict the scatter of

Fisher and Hollomon have pursued this statistical trend in an effort to show the effects of single rate, size, and structure. "The fracture of a real material can be investigated theoretically only by idealizing both the structure of the material and the method of fracture. The idealized model selected for analysis can be justified and accepted as a valid representation if the results of the theoretical analysis are in agreement with the observed facts...."

The above quotation characterizes this work, which again is based on Weibull's fundamental statistical analysis. Their assumptions are:

1. The structure is an elastic solid containing many cracks.
2. The cracks are thin and disc-like with elliptical cross sections.
3. The cracks are orientated at random.
4. The cracks are separated on the average slightly enough so that there is negligible amount of interference of the regions of local distortion which surround each crack under elastic strain.

The author's will not be presented within this paper but it is sufficient to point out that the normal laws of probability and statistical analysis were followed.

The results of these very interesting statistical analyses into brittle fracture give means to calculate the effect of specimen size on the fracture stress, predict the number of

fracture stress measurements (shows when chance causes are operating) and calculate the effects of combined stress and plastic deformation. It is to be remembered that this analysis is based on the assumption that fracture in solid materials is caused by the presence of randomly oriented defects having the properties of cracks. Agreement with known facts, and in particular with the previous work of Griffith is very good. When there is a compressive pre-strain the fracture stress is observed to increase. this is in disagreement with Hollomon and Fisher's work and seriously endangers the micro-crack theory.

Thermodynamic or Energy Theory:-

A number of years ago, ~~Puth~~ Fürth advanced a theory relating the tensile strength of metals to the melting of crystals but there were several serious objections to his work, namely that there were no provisions for the effect of plastic flow, alloying elements, or heat treatment. Another objection was that his theory was impossible to check precisely since it applied only to isotropic bodies that remained completely elastic up to the point of fracture. It is highly doubtful that this is the case since all fractured surfaces show evidence of some plastic flow. Saibel has taken the fundamental idea of relating fracture to the melting phenomenon but he has developed a different relationship that eliminates the major objections to Fürth's work.

Saibel's thermodynamic approach contains three basic

assumptions:-

1. All of the strain energy is available for the abolition of cohesive strength.
2. The heat of fusion is uniformly partitioned throughout the volume occupied by the substance.
3. The quantity of energy required for the abolition of cohesive strength is equal to the fractional change in specific volume as the material passes from the solid state to the liquid, multiplied by the heat of fusion.

Saibel's critical condition is expressed as follows:-

$$u = J L_m \frac{\Delta V}{V}$$

where:-

u is the strain energy / unit volume.

L_m is the latent heat of melting / mol

ΔV is the change in volume of one mol of the substance on passing from the solid to the liquid state.

V is the molecular volume in the solid state at the melting point.

J is the conversion factor to make the units consistent.

It is further noted that the fracture tests are not carried out slowly so there is no leakage of energy in the form of heat out of the specimen and that the change in volume on melting is due to the formation of " holes " as based on the Eyring model of

assumptions:-

1. All of the stored energy is available for the
   abolition of cohesive strength.
2. The heat of fusion is entirely participated
   throughout the volume occupied by the sub-
   stance.
3. The quantity of energy required for the
   abolition of cohesive strength is equal to
   the fractional change in specific volume as
   the material passes from the solid state to
   the liquid, multiplied by the heat of fusion.

Briefly mathical condition is expressed as follows:-

$$\alpha = \gamma L_m \frac{\Delta V}{V}$$

where:-

α is the strain energy / unit volume.

L_m is the latent heat of melting / mol

ΔV is the change in volume of one mol of the sub-
stance on passing from the solid to the liquid
state.

V is the molecular volume in the solid state at
the melting point.

γ is the conversion factor to make the units con-
sistent.

It is further noted that the Lennard-Jones terms are not carried out
formally as there is no leakage of energy in the form of heat out
of the specimen and that the change in volume on melting is due
to the formation of " holes " as based on the Eyring model of

the liquid state.

In using the theory, the following form of the energy equation is useful since it can be considered that, after plastic flow has occured, the elastic energy is negligible, as compared with the energy of distortion, and

$$u = \int \bar{\sigma}\, d\bar{e}$$

where ($\bar{\sigma}$) and ($\bar{e}$) are the significant stress and significant strain. Further, the relation previously noted

$$\bar{\sigma} = k\bar{e}^{n}$$

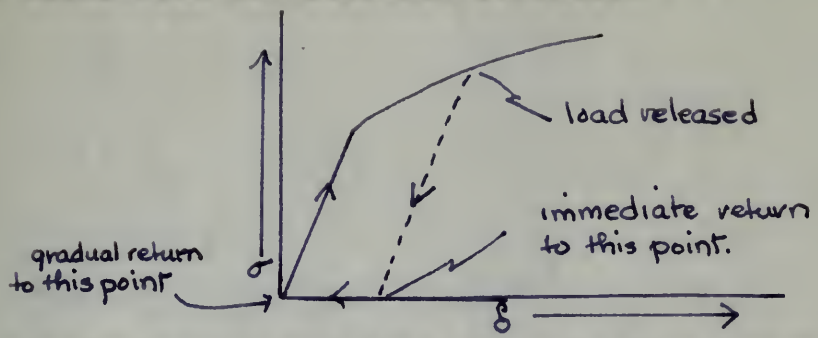
gives a convenient means of actually calculating desired values. Making use of the above noted expressions, Saibel performs calculations for the case of pure brittle fracture and fracture preceded by plastic flow. The fracture stresses (σ_f) as calculated show good agreement with actual values and Saibel states that the agreement is good enough for a purely brittle case and thus no need exists for the micro-crack theory. The discrepencies are due to inaccurate thermodynamic data to a large extent. It is also desired to point out that there is controversy surrounding the important point of whether purely brittle fracture is preceded by some plastic flow. Saibel says that there is always some plastic flow prior to fracture, even if it is only a few atom layers in extent.

The Relaxation Phenomenon:-

It can now be seen that there has been a great difference of opinion and ideas surrounding this problem of brittle fracture. It is a well known fact, however, that the progress of scientific knowledge is measured by the controversial nature of argu-

ments until conclusive and indisputable evidence is presented and accepted. As a further measure of the unsettled state of affairs in this field, Zener has attacked fracture from the standpoint of the relaxation phenomenon, which is in striking contrast to previous ideas.

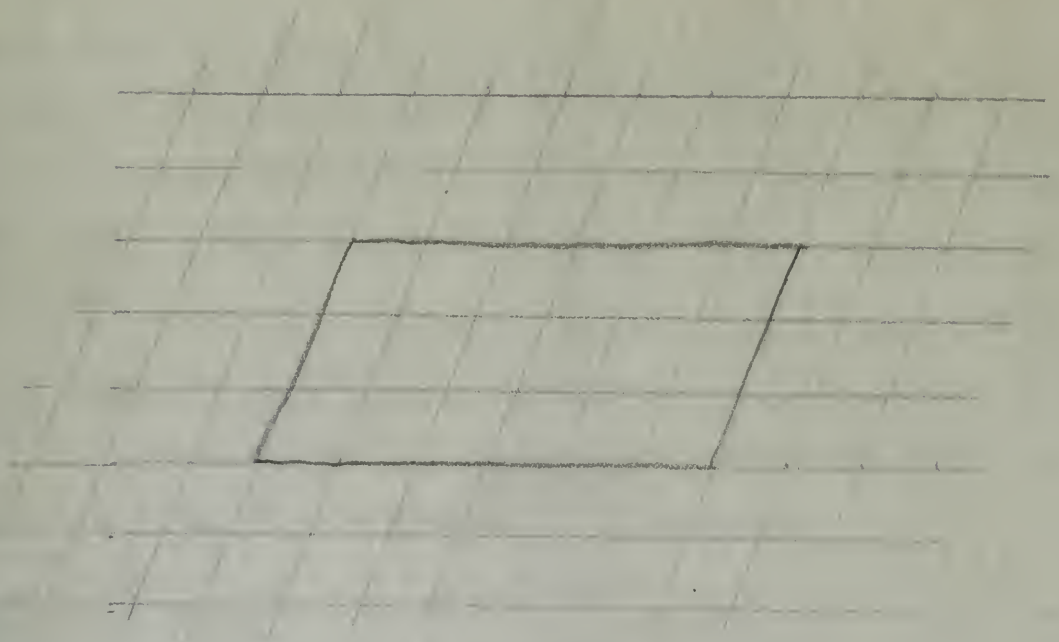
Since relaxation is tied up with anelasticity, the recovery of plastic deformation with time, a true stress-strain diagram will illustrate the over-all effects:-



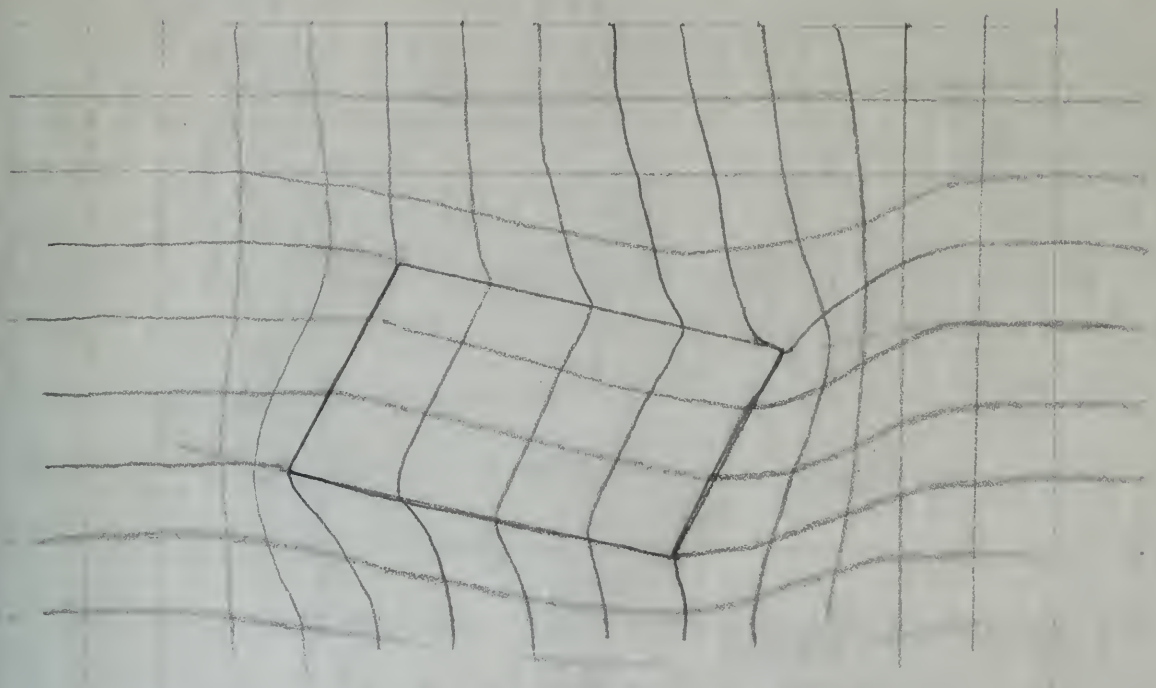
It is to be remembered that elevating the temperature, even slightly, for most metals will eliminate anelastic effects and allow recovery after the stress is removed. It has been believed for a long time that the anelastic effects were due to a region within the metal behaving in a viscuous manner. Grain boundaries and slip bands are the regions which will obey the laws of viscuous behavior. The following sketches serve to show in principle what happens within the crystal. Assume that the heavy lines represent a viscuous slip band within a lattice:

(see next page for sketches)

Load released
immediate return
to this point
gradual return
to this point
δ



Load, no relaxation.



Load removed, no relaxation.

Load, no relaxation.

Load removed, no relaxation.

Note that the stress concentrations at the corners can be very heavy and can only be removed by relaxation or fracture. Zener has used this phenomenon of relaxation to develop a theory which will allow a prediction of the fracture stress.

There is considerable doubt as to whether grain boundaries should be considered at all in attempting to work out a theory of brittle fracture. This is another instance where the controversy is intense and far from being resolved.

Mechanical Testing:-

Since the proof of any theory lies in the effectiveness of its predictions, some mention should be made of the means by which the predictions are tested and their relative merits. Essentially there are four tests;-

1. Simple tension test.
2. Notched tension test.
3. Notch bend test.
4. Notch impact test.

There is also the torsion test which is normally not applied to a study of brittle fracture.

The simple tension test, when used for data on brittle behavior, hardly needs any comment, being familiar in a more or less degree to all metallurgists. It may be noted, in passing, that the complex stress system caused by necking down is ,in general, a negligible factor in brittle fracture but extremely important in analysing ductile behavior.

When the tensile (normal) specimen is notched, a new and very different situation applies. The effect of the notch is two-fold. First, it serves to reduce the cross-section area and, second, it acts as a stress concentrator. There is some controversy over the effectiveness of this type of tensile specimen. The greatest difficulty in the proper interpretation of the data, due to the factors of size effect, stress concentration, and lack of knowledge of the stress state.

Since notches are being used more frequently, a brief explanation of their effect will be useful. Notches are introduced for the purpose of setting up a controlled condition of stress concentration and also to attempt to set up a condition of multi-axial stresses in an effort to study fracture and fracture stress under multi-axial loading. Gensamer has done a great deal of work in attempting to resolve the difficulties and to correlate the data (notched) with each other and also with the simple tensile test. Neuber has developed a series of nomograms which give the stress concentration factors for the elastic state. It is to be noted that the severity of the notch is most important., a sharper notch requiring less energy to initiate a fracture. A stress concentration curve will show the magnitudes of the stresses at the notch. Note that the amplitude falls off rapidly and that the stress level away from the vicinity of the notch is less than if there were no notch. Also note the relative amplitudes of the multi-axial stresses.

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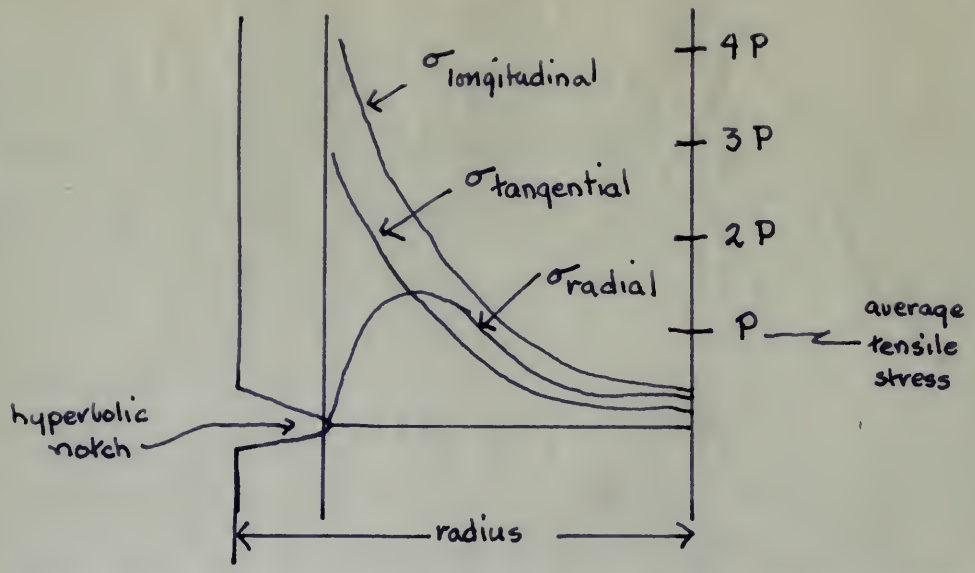
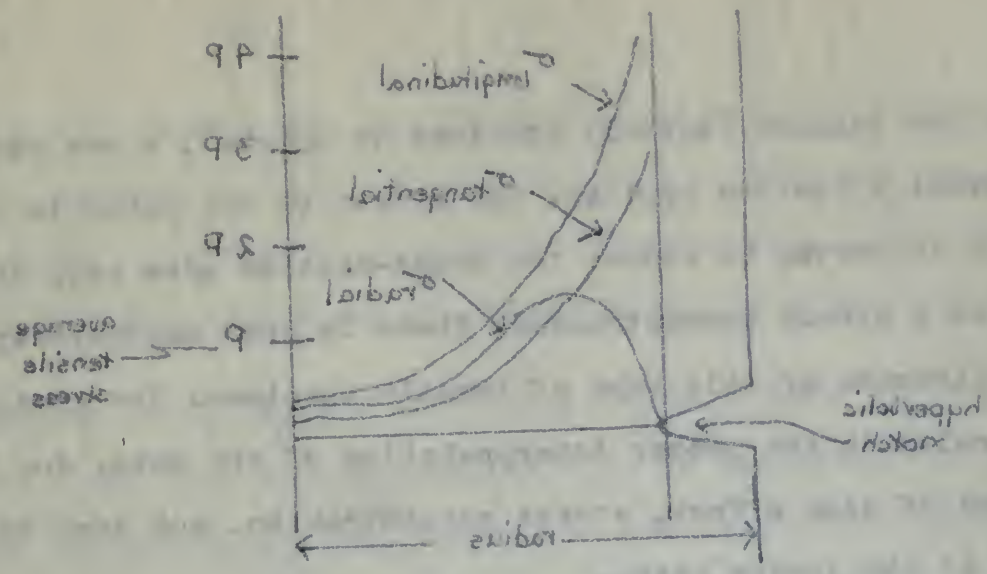


The problem of correlation of the various tests has progressed to a satisfactory state generally and the choice of test will depend upon the relative ability to show desired characteristics.

The notch bend and the notch impact tests are often used when testing brittle behavior, especially the notch impact test. The notch impact test allows for a fast rate of loading not possible in the normal tension test. As will be seen later, this may be an important factor. Due to the extensive work on correlation, confidence in notched tests has increased somewhat.

Influence of Various Factors:-

From time to time, mention has been made of the fact that fracture strength varies according to the influence of various effects. The influence of size on the specimen has already been discussed with the developement of statistical approach by Weibull. In an engineering sense, the effects of various factors can be far more important than the basic theory. Unfortunately the over-all influence of size effect, temperature, strain rate, and other factors can not be well established until the precise


4p
3p
2p
p
Tangential
Radial
Average tensile stress
Hyperbolic notch
radius

mechanism of brittle fracture is fully understood. Despite the latter drawback, a brief discussion of each factor should be made, along with an explanation of the phenomenon when known.

Size Effect;-

Size effect, although mentioned previously, is known to be the lowering of the fracture stress and the energy required to propagate the fracture, when the size of the specimen is increased. This is the most apparent with steel and both Davidenkov and Fisher and Hollomon have used Weibull's statistical methods in an attempt to find a suitable explanation of this phenomenon. Size effect is, by its very nature, closely allied with the distribution of particles, such as carbides, and their shape within the structure of the metal. This is extremely important when considering the design characteristics which are based on the test results of small specimens. There are two separate effects under size effect:

1. The greater statistical probability of finding a defect or significant stress raiser in the microstructure in the larger size.
2. The induced stresses due to restriction of strain in large sizes.(condition of restraint)

The second separate effect is of the utmost importance in the design of structures. Gensamer summed up the difficulties due to size effect by postulating that the size effect is due to inability to scale the fine micro-structure with the specimen size.

Complex Stress System;-

It should be noted that it is extremely difficult, if not impossible, to separate completely the effects, one from another, that the various factors have upon brittle fracture. When one variable is changed usually another will change, but uncontrollably. An attempt, however, will be made to present the facts as best as they are known, keeping in mind the interdependence, which is unavoidable.

Complex stresses are currently the subject of a great deal of work in which McAdam has been prominent. In a broad way, it is known that biaxial and triaxial stresses will reduce the ductility of metals. The effect on fracture stress is still being argued but it seems that the fracture stress is raised when triaxiality is increased. This fact has been borne out by McAdam and by Sachs. Perhaps the state of affairs can be partially understood when it is realized that conditions of multi-axial stress are obtained ~~by~~ either by notches or by some attempt at direct biaxial stress systems, using clamped down diaphragms or tubes, pulled in tensile machines. Comparisons, particularly in investigating ductility, are often made with normal, un-notched tensile specimens. An interesting postulation made by Jelinek is that there may be two criteria for the fracture of a single metal; maximum shear stress and maximum tensile stress. Fracture would occur when the most severe condition is reached. This is supposed to explain the behavior of cast iron, but then it is to be remembered that Sternberg explained the same baffling behavior of cast iron by a non-linear form of Hooke's law. This is sufficient to show that there

Complex Stress Systems-

It should be noted that it is extremely difficult, if not impossible, to experimentally duplicate complex stress systems, such as various fracture data upon brittle materials. When one variable is changed usually another will change, but unpredictably. An attempt, however, will be made to present the facts as they are known, keeping in mind the interdependence, which is encountered.

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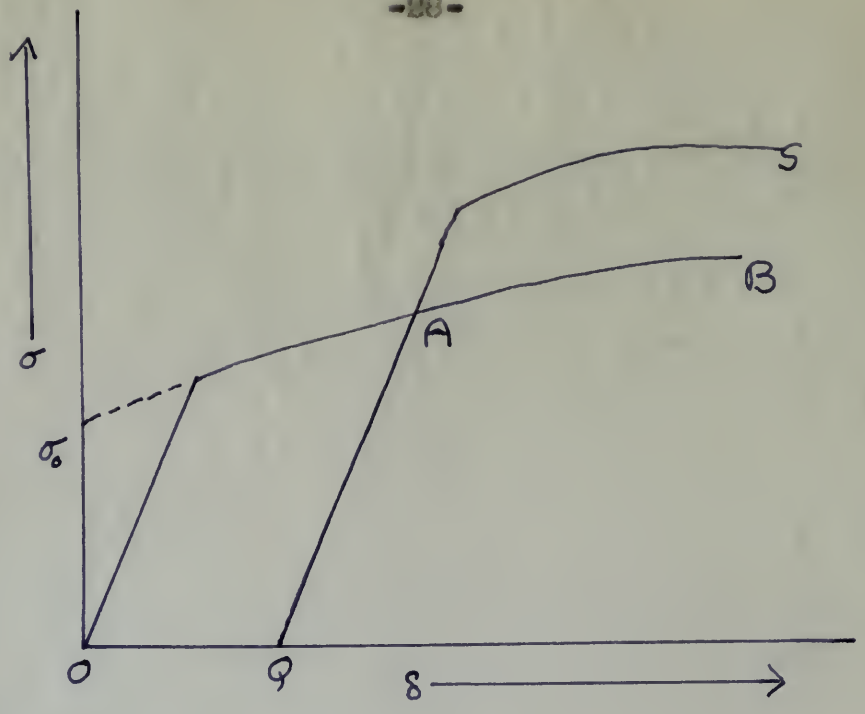
is little or no reliable data upon which to base any definite conclusions, other than the apparent reduction of ductility under complex stresses.

Effect of Prior Strain;-

There have been several fairly recent investigations into the effect of prior strain on the fracture strength. Sakharov, in Russia, and Hollomon and Zener, here, have shown that, in the particular case of pearlitic steel, the tensile fracture stress must decrease with the first small amount of deformation. This is shown by the specimen fracturing at a lower stress than the yield stress, In addition to this effect, their work corroborates the early postulation of a fracture curve, as made by Ludwik. The stress required for fracture was shown to increase with increasing strain. Bridgeman applied hydrostatic pressure to a tensile specimen under tension and it was found that the fracture stress was raised. His work tends to substantiate the micro-crack theory of fracture in that there may be a sealing up or welding up of the microcracks under the hydrostatic pressure which would, according to that theory, raise the fracture strength. Saibel has explained the effect of prior strain on fracture from the standpoint of his thermodynamic theory of fracture. His results agree well with the meager experimental data available. The stressing cycle used by Saibel is noted on the next page:-



1. metal is pre-strained a given amount at room temperature. (OAQ)
2. the temperature is lowered.
3. the specimen is pulled to fracture. (QARS)

The final status of knowledge to date allows for a prediction of the effect of prior strain by use of the micro-crack theory and energy considerations along with the effects of micro-structure.

Effect of Cyclic Loading:-

The question of the effects of cyclic loading or fatigue has been studied from the engineering viewpoint. There is a great deal of data available on fatigue strengths and endurance limits but very little of it delves into the fundamentals of the situation. A little work has been done relative to the dissipation of heat produced as a result of the alternating straining within the metal. This may tie in with the current theories regarding the viscuous regions existing at grain boundaries and within slip bands to give a coherent picture. Another unresolved effect is that of prestraining on the fatigue strength. There

(OAQ)

2. under is pre-stressed a given amount at room temperature.

3. the temperature is lowered.

(QABC)

5. the steering is pulled to tension.

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are some claims that fatigue is merely strain hardening up to the fracture point. Taken as a whole, it can be seen that there is much work yet to be done on this particular factor. The over-all effect is, however, to reduce the fracture strength as the number of cycles increase

Effect of Strain Rate:-

Strain rates, or the dynamic conditions involved, have come in for a great deal of attention. There are two widely divergent strain rates of interest, namely that rate just above being static and rates approaching ballistic velocities. As a side line to the research on strain rates, some interesting work has been done on the strain wave propagation through metals. Briefly, it has been found that the effect of strain rate at normal speeds of testing are not significant and are unimportant. When strain rates approaching ballistic velocities are encountered, that strain rate is very important, since the increase in the flow stress may make it approach the fracture stress and thus cause failure. The elastic waves may also reach fracture magnitudes ahead of the plastic strain waves and thus cause brittleness.

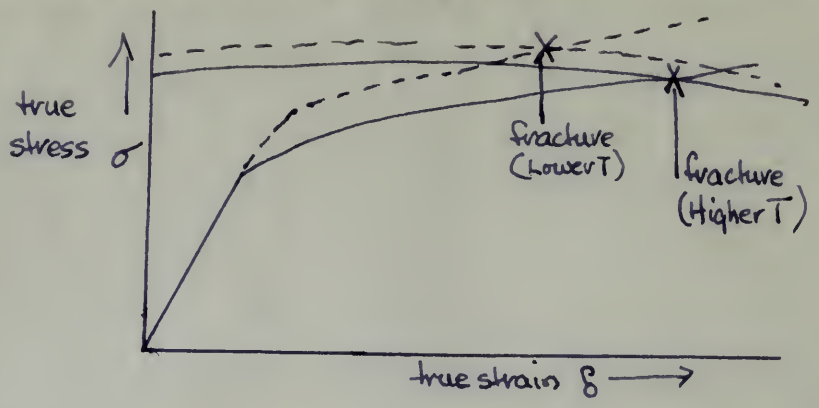
Hollomon and Zener have attempted to derive an interrelation between strain rate and temperature as they affect fracture strength. It is known that decreasing the temperature and raising the strain rate increase the stress required for fracture. This brings up the question of applicability of the mechanical equation of the state of brittle fracture as well as to ductile failure or flow. This particular correlation for the case of brittle fracture is

not as well established as it is for flow.

Effect of Temperature;-

A most important factor regarding fracture strength is the temperature of fracture. It is significant to note that a single metal, such as pearlitic steel, can be made to behave in either a ductile or brittle manner solely by temperature variation. Early work by the Russians attempted to demonstrate that the fracture strength was independent of temperature, but they neglected the important factor of strain rate, which has been discussed above. Present work has shown that fracture strength increases with a decrease in temperature. Departing a moment from purely brittle behavior, it is desired to point out that the flow stress decreases as the temperature is raised until, at the melting point, there is no yielding stress. The flow stress increases with a reduction in temperature but at a more rapid rate than the fracture stress, which allows for fracture at less deformation. This can be shown graphically on a true stress-strain curve. It is interesting to note that the type of fracture changes from a transcrystalline to intercrystalline type as the melting temperature is approached. Hollomon reports a similar behavior at varying strain rates; fast strain rates producing transcrystalline fractures. There is little or no work in the field of combined stresses relative to the effect of temperature on brittle fracture and, in general, more work should be done to try to isolate completely the temperature effects and eliminate the controversies

beyond a doubt. The following chart shows the over-all effect of temperature as currently accepted:



These curves also serve to illustrate how any factor that affects either the flow or fracture curve will affect the ductility and, consequently, the brittle behavior. It is to be remem bered that some factors move both curves in the same direction but at different rates. Thus it can be seen that the true Stress-strain curves are very helpful in studying over-all effects, especially for engineering application.

Effect of Structure;-

The effect of structure has already been touched upon at several points, mostly during the discussion of statistical analysis. In addition to the carbide particles mentioned, the precipitate formed in age-hardening alloys may have a somewhat similar effect since an embrittling is known to occur in these alloys. A martensitic structure possesses a very high fracture strength, even at low temperatures, when it is tempered and it is generally recognized as being the most desirable structure in steels where low temperature impact properties are required. The effect of alloying

beyond a limit. The following short comes the over-all effect of temperature as commonly accepted:

There curves also serve to illustrate how any factor that affects the flow or fracture curve will affect the ductility and, consequently, the metallic behavior. It is to be seen that some factors move both curves in the same direction but at different rates. Thus it can be seen that high-temperature curves are very helpful in studying over-all effects, especially for explaining application.

Effect of Structure:-

The effect of structure has already been touched upon at several points, mostly during the discussion of statistical analysis. In addition to the marble particles mentioned, the precipitate formed in age-hardening alloys may have a measured influence either on precipitating in known to occur in those alloys. A heterogeneous structure possesses a very high fracture strength, even at low temperature, when it is textured and it is generally recognized as being the most desirable structure in steels where low temperatures are anticipated. The effect of alloying

elements should not be overlooked, since it is well known that solid solution hardening has a definite effect in raising the fracture strength. Nickel is somewhat beneficial in aiding low temperature impact strength. Again, more good work is required along these lines to attempt to pin down the influence of structure (and composition) from a fundamental basis.

Restatement;-

This paper has followed the problem of brittle fracture in metals from the logical beginnings in mechanical metallurgy, where the basic engineering mathematical expressions were shown through several controversial subjects (theories) and, finally, the effects of various factors on the fracture strength were briefly reviewed. It depends on one's viewpoint, ie: research or engineering, to determine which part of the discussion is most applicable. That the subject is important is an acknowledged fact and it is all too apparent that there is a great deal more work to be done along all phases of this problem. Fortunately the work is being carried forward.

It should also be pointed out that, at this time, it cannot be said which theory is the precisely correct one or what is the correct mechanism of brittle fracture. There is too much conflicting information and it must be remembered that the correct theory must agree accurately with observed facts and, furthermore, must be able to predict behavior based on a few experimental

calculations. No theory yet presented can fulfill all the requirements. However,each theory and investigation, no matter how small, has added something to the general overall picture. It is just a matter of time and effort until all the pieces of the puzzle will fall into place. When the theory is complete and known as a"law of brittle fracture", the engineering benefits will be immense and vastly important.

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